BFKL, BK and the Infrared

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Abstract. The perturbative non-linear (NL) effects in the small-x evolution of the gluon densities depend crucially on the infrared (IR) regularization. The IR regulator, R_c, is determined by the scale of the non-perturbative fluctuations of QCD vacuum. From the instanton models and from the lattice $R_c \sim 0.3$ fm. For perturbative gluons with the propagation length $R_c = 0.26$ fm the linear BFKL gives a good description of the proton structure function $F_2(x,Q^2)$ in a wide range of x and Q^2. The NL effects turn out to be rather weak and amount to the 10% correction to $F_2(x,Q^2)$ for $x \lesssim 10^{-5}$. Much more pronounced NL effects were found in the non-linear model, described in the literature, with a very soft IR regularization corresponding to the IR cutoff at $\simeq \Lambda_{QCD}^{-1}$. The latter issue is also commented below.

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Introduction

The non-perturbative fluctuations of the QCD vacuum [1] restrict the phase space for the perturbative (real and virtual) gluons of the BFKL cascade [2] thus introducing a new scale: the correlation/propagation radius R_c of perturbative gluons. From the fits to lattice data on field strength correlators $R_c \simeq 0.3$ fm [3]. The effects of finite R_c are consistently incorporated by the color dipole (CD) BFKL equation [4, 5]. The perturbative gluons with short propagation length do not walk to large distances, where they supposedly fuse together [6]. The fusion probability appears to be controlled by the dimensionless parameter $R_c^2/8B$, where B stands for the diffraction cone slope [7, 8].

In this communication we discuss the BFKL [2], BK [6] phenomenology of DIS in presence of finite correlation length R_c with particular emphasis on a sharpened sensitivity of the non-linear effects to the infrared.

Color screening of BFKL gluons

The finite correlation length implies the Yukawa screened transverse chromo-electric field of the relativistic quark,

$$\vec{\mathscr{E}}(\vec{\rho}) \sim g_S(\rho) K_1(\rho/R_c)\vec{\rho}/\rho, \tag{1}$$

where ρ is the $q-g$ separation. The kernel $\mathscr{K}$ of the CD BFKL equation for the color dipole cross section,

$$\partial_\xi \sigma(\xi,r) = \mathscr{K} \otimes \sigma(\xi,r),\ \xi = \log(1/x), \tag{2}$$

is related to the flux of the Weizsäcker - Williams gluons $|\vec{\mathscr{E}}(\vec{\rho}) - \vec{\mathscr{E}}(\vec{\rho}+\vec{r})|^2$ radiated by the relativistic $q\bar{q}$-dipole $\vec{r}$ [4, 5] . The asymptotic freedom dictates that $\vec{\mathscr{E}}(\vec{\rho})$ must be calculated with the running QCD charge $g_S(\rho) = \sqrt{4\pi\alpha_S(\rho)}$ and $\alpha_S(\rho) = 4\pi/\beta_0 \log(C^2/\rho^2\Lambda^2_{QCD})$, where $C = 1.5$.

DGLAP ordering of dipole sizes and the infrared

Eq.(2), with the BK non-linearity [6] included, greatly simplifies for the DGLAP ordering of dipole sizes, $r^2 \ll \rho^2 \ll R_c^2$,

$$\begin{aligned} \partial_\xi \sigma(\xi,r) = \frac{C_F}{\pi}\alpha_S(r) r^2 \int_{r^2}^{R_c^2} \frac{d\rho^2}{\rho^4} \times \\ \times \left[2\sigma(\xi,\rho) - \frac{\sigma(\xi,\rho)^2}{8\pi B}\right]. \end{aligned} \tag{3}$$

Our definition of the profile function in the impact parameter b-space is

$$\Gamma(\xi,r,\mathbf{b}) = \frac{\sigma(\xi,r)}{4\pi B(\xi,r)} \exp\left[-\frac{b^2}{2B}\right], \tag{4}$$

where B is the diffraction cone slope and $d\sigma_{el}/dt \sim \exp[Bt]$. Eq.(4) implies that the unitarity limit for σ is $8\pi B$.

The diffraction slope for the forward cone in the dipole-nucleon scattering was presented in [9] in a very symmetric form

$$B(\xi,r) = \frac{1}{2}\langle b^2\rangle = \frac{1}{8}r^2 + \frac{1}{3}R_N^2 + 2\alpha'_{\mathbf{IP}}\xi. \tag{5}$$

The dynamical component of B is given by the last term in Eq. (5) where $\alpha'_{\mathbf{IP}}$ is the Pomeron trajectory slope evaluated first in [10] (see also [9]). Here we only cite the order of magnitude estimate [9]

$$\alpha'_{\mathbf{IP}} \sim \frac{3}{16\pi^2} \int d^2\vec{r}\ \alpha_S(r) R_c^{-2} r^2 K_1^2(r/R_c) \sim \frac{3}{16\pi}\alpha_S(R_c) R_c^2, \tag{6}$$

which clearly shows the connection between the dimensionful $\alpha'_{\mathbf{IP}}$ and the non-perturbative infrared parameter R_c. In Eq. (5) the gluon-probed radius of the proton is a phenomenological parameter to be determined from the experiment. The analysis of Ref. [11] gives $R_N^2 \approx 12\,\mathrm{GeV}^{-2}$.

The function $\rho^{-2}\sigma(\xi,\rho) \sim \alpha_S(\rho)G(x,\rho)$, where G is the integrated gluon density, is flat in ρ^2 and the non-linear term in Eq.(3) is dominated by $\rho \sim R_c$:

$$\frac{1}{8B}\int_{r^2}^{R_c^2}\frac{d\rho^2}{\rho^4}\sigma(\xi,\rho)^2 \simeq \frac{R_c^2}{8B}\left(\frac{\pi^2}{N_c}\alpha_S(R_c)G(x,R_c)\right)^2 .$$

Thus, the small parameter $R_c^2/8B$ enters the game. To see it in action a partial solution to Eq.(3) is needed.

Partial solution to GLR-MQ

The differential form of Eq.(3) for G is the GLR-MQ equation [12]

$$\partial_\xi \partial_\eta G(\xi,\eta) = cG(\xi,\eta) - a(\eta)G^2(\xi,\eta), \tag{7}$$

where $c = 8C_F/\beta_0$, $\eta = \log\left[\alpha_S(R_c)/\alpha_S(\rho)\right]$,

$$a(\eta) = a(0)\exp[-\eta - \lambda(e^\eta - 1)], \tag{8}$$

$a(0) = \alpha_S(R_c)\pi R_c^2/4\beta_0 B$ and $\lambda = 4\pi/\beta_0\alpha_S(R_c)$. We solve Eq.(7) making use of the exact solution of

$$\partial_\xi \partial_\eta G(\xi,\eta) = cG(\xi,\eta)$$

as a boundary condition. To simplify Eq.(7) we substitute the steeply falling function $a(\eta)$ with $a(\eta) = a(0)\exp\left[-\gamma\eta\right]$. The partial solution of the simplified problem is found readily:

$$G(\xi,\eta) = \frac{\exp\left(\Delta\xi + \gamma\eta\right)}{\omega\exp\left(\Delta\xi\right) + const}. \tag{9}$$

Here $\gamma = c/\Delta$ with asymptotic value $\Delta = 0.4$ and

$$\omega = \frac{a(0)}{c} = \frac{R_c^2}{8B}\cdot\frac{\pi\alpha_S(R_c)}{2N_c}. \tag{10}$$

One can see that the gluon fusion mechanism tames the exponential ξ-growth of $G(\xi,\eta)$ but fails to stop the accumulation of large logarithms, $\eta = \log(1/\alpha_S(r))$, at very small $r^2 \ll R_c^2$.

For vanishing non-linearity, $\omega \to 0$, Eq.(9) matches the exact solution to the CD BFKL equation found in Ref.[13]

$$G \sim \exp(\Delta\xi + \gamma\eta) \sim \left(\frac{1}{x}\right)^\Delta \left[\frac{1}{\alpha_S(r)}\right]^\gamma. \tag{11}$$

In the limit $\xi \to \infty$ the gluon density saturates,

$$G \sim \omega^{-1}e^{\gamma\eta}, \tag{12}$$

and the corresponding dipole cross section reads

$$\sigma(r) = 8\pi B\cdot\frac{r^2}{R_c^2}\cdot 2\left[\frac{\alpha_S(R_c)}{\alpha_S(r)}\right]^\gamma. \tag{13}$$

Regime of the additive quark model

At large $r \gtrsim R_c$ a sort of the additive quark model is recovered: the (anti)quark of the dipole $\vec{r}$ develops its own perturbative gluonic cloud and the pattern of the gluon fusion changes dramatically [8]. From Ref.[8] it follows that the the non-linear correction to the dipole cross section is

$$\delta\sigma \sim R_c^{-2} \int^{R_c^2} d\rho^2 K_1^2(\rho/R_c) \frac{\sigma(\xi,\rho)\sigma(\xi,r)}{8\pi B} \sim \\ \sim \frac{\sigma(\xi,R_c)\sigma(\xi,r)}{8\pi B}. \quad (14)$$

For R_c much smaller than the nucleon size $\sigma(R_c) \propto R_c^2$. Therefore, the magnitude of non-linear effects is controlled, like in the case of small dipoles, by the ratio $R_c^2/8B$.

Small R_c - weak non-linearity

In Ref.[8] we solved numerically the BFKL and BK equations with purely perturbative initial conditions and the IR regularization described above. Our finding is that the smallness of the ratio $R_c^2/8\pi B$ makes the non-linear effects rather weak even at the lowest Bjorken x available at HERA. The linear BFKL with the running coupling and the infrared regulator $R_c = 0.26$ fm gives a good description of the proton structure function $F_2(x,Q^2)$ in a wide range of x and Q^2 [8]. For the smallest available $x \lesssim 10^{-5}$ the 10% NL correction improves the agreement with data, though.

Soft IR regularization - fully developed non-linearity

In Ref.[14] the non-linear BK-analysis of HERA data on $F_2(x,Q^2)$ was presented. The infrared cutoff for the purely perturbative BK-kernel is deep in the non-perturbative region,

$$r_{IR} = 2/Q_s \simeq 4\,\mathrm{GeV}^{-1} \simeq \Lambda_{QCD}^{-1}, \quad (15)$$

and the non-linear effects are absolutely important to tame the rapid growth of the linear term in the BK equation. The running strong coupling $\alpha_S(r)$ at $r = r_{IR}$ in [14] appears to be surprisingly small, $\alpha_S(r_{IR}) \simeq 0.45$, thus assuming an applicability of perturbative QCD to arbitrarily large distances $\sim \Lambda_{QCD}^{-1}$. However, it is well known that the non-perturbative fields form structures with sizes significantly smaller than Λ_{QCD}^{-1} and local field strength much larger than Λ_{QCD}^2. Instantons are one of them [1]. Direct confirmation of this picture comes from the lattice [3]. Therefore, the approach developed in [14] may lead to a very good BK-description of the HERA data but does not agree with the current understanding of what is perturbative and non-perturbative in hadronic physics.

Summary

It is not surprising that introducing a small propagation length for perturbative gluons pushes the nonlinear effects to very small x. More surprising is that within the linear CD BFKL approach this small length, $R_c = 0.26$ fm, results in the correct x-dependence of $F_2(x,Q^2)$ in a wide range of x and Q^2. The 10% non-linear BK-correction improves the agreement with HERA data at smallest available x [8].

The non-linear BK-description [14] of HERA data which extends perturbative QCD to distances $\simeq \Lambda_{QCD}^{-1}$ contradicts to well established non-perturbative phenomena and apparently breaks the hierarchy of scales of soft and hard hadronic physics.

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